Global Real Rates: A Secular Approach by Pierre-Olivier Gourinchas and Helene Rey

Discussion by Sebnem Kalemli-Ozcan

This paper asks two very important and related questions: Why do real risk free rates decline and how long they will stay low? Differently than the existing literature on the topic, the authors adopt a present value approach to decompose fluctuations in global consumption-to-wealth ratio over long periods of time. This decomposition involves three components: risk free rate, risk premia, and consumption growth. The authors write down a model to analyze role of shocks on each of these components to discover the underlying deep causes of fluctuations in consumption-to-wealth ratio. They undertake such an exercise since consumption-to-wealth ratio predicts risk free rates. Their main results are as follows:

1. Consumption-to-wealth ratio is a strong predictor of risk-free rates, term premium, and population growth.
2. Macro shocks and financial shocks both have a role in explaining real risk free rates.
3. Risk free rates will stay low for an extended period of time.
4. Overall the results suggest a decline in *natural* interest rate.

This is an excellent paper with thought provoking results. In my comments, I will try to clarify certain issues to make the interpretation of the results sharper. The first issue is on measurement of real risk free rates and relation to natural rate. Real rate is the sum of real risk free rate and risk premium. And natural rate is only equal to real rate when real rate is the one that equates output to potential output. Put it differently, real rate only equals to natural rate under monetary policy neutrality. Hence, a decline in real risk free rate may or may not suggest a decline in natural rate since this will also depend on risk premium and monetary policy. In this juncture, it is important how to measure risk free real rates. The authors use a measure of nominal rates on 3 month treasury bills after subtracting CPI inflation. Maybe a better measure is nominal rates minus inflation expectations and/or yields on inflation-indexed bonds.

The next issue is on the key cause of decline in real rates. The existing literature takes two opposing views. The "investment view" or the "savings view." The investment view, associated with Larry Summers, argue that the decline in real rates is due to a decline in investment due to lack of good investment opportunities given the lower relative price of investment. The savings view can have two different components. The first one, associated with Ben Bernanke, is about savings glut in the rest of the world due to demographic changes and those savings are invested in US risk free assets, lowering their yield. The second component of savings view is about debt accumulation and associated deleveraging that can take a long time, as argued by Carmen Reinhart and Ken Rogoff. There can also be a twist to this story in terms of preferences where investors prefer risk free assets to risky ones so savings are channelled to risk free assets, that leads to a decline in risk free rate and an increase in risk premium. The early proponents of this view are Ricardo Caballero, Emmanuel Farhi and Pierre-Olivier Gourinchas.

The current paper is different than all the other papers given their long-run historical approach. However, it is similar to the papers that support the savings view since the current paper's long-run approach makes it clear that savings especially after the financial crises have a big role in explaining declining real risk free rates. The problem is that, it is not fully clear if increases in savings after big financial crises is the only force behind the declining real risk free rates or there are also other factors at play? For example, savings might be higher due to demographics changes and not due to deleveraging effect of financial crises.

The authors realize this and run predictive regressions to sort this out. Their predictive regressions regress several outcome variables, namely, risk free rates, consumption growth, equity premium, population growth, term premium, and credit growth, on consumption-to-wealth ratio and find that this ratio can predict risk free rates, term premium, and population growth at long horizons. Revisiting these results suggest that a *stationary* consumption-to-wealth ratio, as assumed by their decomposition approach, can only predict risk free rates and term premium and not population growth. Checking for stationary of the consumption-to-wealth ratio and adding a trend to the predictive regressions deliver these results as shown in Tables 1 and 2. A horse race predicting regression in Table 3, shows that, it is not only consumption-to-wealth ratio but also term premium can predict risk free rates.

The final issue is on identification. The decomposition of consumption-to-wealth ratio does not have a causal interpretation. But, we want to know what causes the fluctuations in consumption-to-wealth ratio. Again, realizing this fully, the authors run several different exercises, where each delivers a different result. For example, the VAR analysis says risk premium is not important for consumption-to-wealth ratio. But, the OLS says risk/term premium is very important for consumption-to-wealth ratio. The VAR says productivity shocks and demographic shocks seem to be more important than deleveraging shock but simple plots of data seems to suggest a bigger role for deleveraging and risk appetite shocks. The key reason for this dilemma is the fact that the model based VAR forces the Euler equation to hold and hence there is a negative association between the risk free rate component and consumption component, whereas a deleveraging shock implies a positive association between the risk free rate and the consumption growth component.

I propose that the authors can further delve into this issue by using their model to identify the effect of shocks on consumption-to-wealth ratio and risk free rates. They have four different shocks, that are a productivity shock, a demographic shock, a deleveraging shock, and a risk appetite shock. They investigate each separately using a reduced form VAR but if they evaluate all together instead, pushing their structural model further then the most important determinant of consumption-to-wealth ratio can survive. This can be done by adding all the shocks to the model and calculating the share of variation explained by each shock. Undertaking this exercise shows that deleveraging shock explains consumption-to-wealth ratio and deleveraging shock together with the risk appetite shock explain risk free rate. Table 4 shows the fit of such a model is good. Table 5 shows that deleveraging shock and risk appetite shock explain about 31 and 63 percent of risk free rate movements, respectively. Productivity shock only explains 6 percent of risk free rate fluctuations and the effects of demographic shock are negligible. On the other hand, consumption to wealth ratio is mainly explained by deleveraging shock (92 percent). It implies that deleveraging shock primarily increases the correlation between risk free rate and consumption to wealth ratio. Appendix provides details on the model with all the shocks.

In closing, I want to re-emphasize that this is an important contribution showing effects of savings increases as a result of debt super cycle and deleveraging on real risk free rate decline. There are also important policy implications: First, the result that term premium can predict short-term risk free rates, suggest an important role of expectations, which is essential for monetary policy making. The second important policy implication is how persistent the effects of debt driven financial crises on risk free rates can be and the final one is on the effectiveness of monetary policy since under persistent low interest rates, this will be in doubt.

Table 1: Testing for Stationarity

Null hypothesis: The variable has a unit root				
Variable	Sample	Specification†	t-statistic	p-value
U.S. ln(C/W)	1870 - 2015	No intercept and trend	0.534	0.830
		Intercept only	-2.592	0.097
		Intercept and trend	-3.430	0.052
	1920 - 2015	No intercept and trend	0.629	0.851
		Intercept only	-1.173	0.683
		Intercept and trend	-1.303	0.881
G-4 ln(C/W)	1920 - 2015	No intercept and trend	0.876	0.897
		Intercept only	-0.862	0.796
		Intercept and trend	-1.123	0.919

The equation for the augmented Dickey-Fuller test is specified as $\Delta y_t = \gamma y_{t-1} + \sum_{s=1}^{k} \delta_s \Delta y_{t-s} + c + \beta t + \epsilon_t$. Reject unit root and establish stationarity only for 1870-2015 for US.

Table 2: Predictive Regressions With Trend

United States (1870 - 2015)

Forecast Horizon	1	2	5	10	1	2	5	10
	(1) No Trend				(2) With Trend			
A. Short term interest rate								
$ln(C/W)_t$	0.13**	0.14**	0.14***	0.15***	0.09	0.10	0.12**	0.13***
	(0.06)	(0.06)	(0.04)	(0.03)	(0.08)	(0.08)	(0.05)	(0.03)
R^2	[0.09]	[0.11]	[0.19]	[0.29]	[0.10]	[0.13]	[0.21]	[0.30]
B. Consumption Growth (per capita)								
$ln(C/W)_t$	-0.03	0	0.01	-0.01	-0.03	0.01	0.03	0
	(0.03)	(0.03)	(0.03)	(0.02)	(0.04)	(0.04)	(0.03)	(0.02)
R^2	[0]	[0]	[0]	[0]	[0]	[0]	[0.03]	[0.07]
C. Equity Premium								
$ln(C/W)_t$	0.13	0.12	0.01	-0.04	0.29	0.26	0.09	-0.01
	(0.15)	(0.15)	(0.09)	(0.07)	(0.19)	(0.19)	(0.10)	(0.07)
R^2	[0]	[0]	[0]	[0]	[0.01]	[0.03]	[0.02]	[0.02]
D. Population Growth								
$ln(C/W)_t$	0.03***	0.03***	0.03***	0.02***	0.01	0.01	0.01*	0.01*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	[0.30]	[0.32]	[0.34]	[0.31]	[0.62]	[0.64]	[0.67]	[0.68]
E. Term Premium								
$ln(C/W)_t$	-0.05***	-0.05***	-0.05***	-0.04***	-0.03	-0.03*	-0.03***	-0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)
R^2	[0.11]	[0.15]	[0.27]	[0.27]	[0.17]	[0.23]	[0.40]	[0.52]

U.S., U.K., France and Germany (1920 - 2015)

A. Short term interest rate								
$ln(C/W)_t$	0.07	0.08	0.12***	0.17***	0.07	0.08	0.13***	0.17***
	(0.05)	(0.05)	(0.04)	(0.04)	(0.06)	(0.06)	(0.05)	(0.04)
R^2	[0.03]	[0.05]	[0.18]	[0.35]	[0.02]	[0.04]	[0.17]	[0.35]
E. Term Premium								
$ln(C/W)_t$	-0.03**	-0.04**	-0.05***	-0.04***	-0.03	-0.03**	-0.04***	-0.04***
	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)
R^2	[0.07]	[0.12]	[0.36]	[0.38]	[0.09]	[0.14]	[0.40]	[0.44]

Table 3: Horse-Race Predictive Regressions

United States (1870 - 2015)

Horizon	1	2	5	10	1	2	5	10
	No C/W and other variables				All variables			
$ln(C/W)_t$					0.06	0.08	0.09**	0.11***
					(0.06)	(0.07)	(0.05)	(0.03)
C. growth $_t$	-0.02	-0.05	0.03	0.00	-0.04	-0.07	0.01	-0.02
	(0.13)	(0.10)	(0.06)	(0.06)	(0.13)	(0.09)	(0.05)	(0.04)
EP_t	-0.01	0.02	0.03*	0.01	0.00	0.03	0.03*	0.02*
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)
Pop. growth_t	1.34	1.25	1.69*	1.83**	0.77	0.50	0.81	0.94
	(1.13)	(1.13)	(0.93)	(0.83)	(1.36)	(1.39)	(1.12)	(0.81)
TP_t	-1.22***	-1.20***	-0.81***	-0.58***	-1.17***	-1.13***	-0.73***	-0.47**
	(0.36)	(0.39)	(0.27)	(0.20)	(0.36)	(0.40)	(0.27)	(0.19)
R^2	[0.21]	[0.22]	[0.24]	[0.27]	[0.21]	[0.25]	[0.30]	[0.38]

U.S., U.K., France and Germany (1920 - 2015)

Horizon	1	2	5	10	1	2	5	10
	No C/W and other variables				All variables			
$ln(C/W)_t$					0.02	0.04	0.10**	0.14***
					(0.04)	(0.04)	(0.04)	(0.03)
C. growth $_t$	-0.05	0.01	0.16	0.17*	-0.07	-0.03	0.06	0.05
	(0.16)	(0.15)	(0.14)	(0.09)	(0.15)	(0.13)	(0.13)	(0.09)
EP_t	-0.01	0.01	0.01	-0.01	-0.01	0.01	0.01	0.00
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Pop. growth_t	1.01	1.18	1.66*	1.32*	0.91	0.80	0.72	0.15
	(1.00)	(1.16)	(0.93)	(0.74)	(1.15)	(1.20)	(0.93)	(0.60)
TP_t	-1.42***	-1.49***	-1.00**	-0.80***	-1.40***	-1.45***	-0.93**	-0.65***
	(0.34)	(0.36)	(0.39)	(0.29)	(0.34)	(0.37)	(0.37)	(0.24)
R^2	[0.24]	[0.27]	[0.20]	[0.19]	[0.24]	[0.28]	[0.28]	[0.41]

Table 4: Model Fit

Data Risk Free Rate Predicting (U.S., 1870 - 2015)				
Forecast Horizon (Years)				
	1	2	5	10
ln(C/W)	0.13**	0.14**	0.14***	0.15***
R^2	[0.09]	[0.11]	[0.19]	[0.29]

Model Risk Free Rate Predicting				
Forecast Horizon (Years)				
	1	2	5	10
ln(C/W)	0.11***	0.11***	0.10***	0.08***
R^2	[0.20]	[0.23]	[0.25]	[0.22]

Table 5: Using Model for Identification

	U.S. (1870 - 2015), Contribution of each shock (percent)			
	Productivity (g)	Demographics (n)	Deleveraging (ρ)	Risk App. (θ)
ln(C/W)	2.18	1.34	92.01	4.47
Risk free rate	5.80	0.24	30.58	63.38

The table reports the share of unconditional variance of log consumption to wealth (C/W) and risk free rate explained by each shock. The share of productivity and population growth shocks includes both first and second moment shock.

1 Appendix: Model

Adopt the model in the section 3.1.4 of Gourinchas and Rey (2018). The model assumes Epstein-Zin recursive preference of representative household. Add productivity (g), demographics (n), deleveraging (ρ), and risk appetite (θ) shock together in the model. Notations are the same as in Gourinchas and Rey (2018), unless stated otherwise. Using the fact that consumption growth ΔlnC_{t+1} equals the sum of consumption per capita growth (g_{t+1}) and population growth (n_{t+1}), one can obtain the Euler equation for the risk free rate as follows:

$$r_t^f = \rho_t + \sigma E_t[g_{t+1} + n_{t+1}] + \frac{\theta_t - 1}{2}\sigma_{r,t}^2 - \frac{\theta_t \sigma^2}{2}(\sigma_{g,t}^2 + \sigma_{n,t}^2 + 2cov_t(g_{t+1}, n_{t+1})) \tag{1.1}$$

where $cov_t(g_{t+1}, n_{t+1})$ is assumed to be small and constant for simplicity.

The budget constraint is:

$$lnC_t - lnW_t = \rho_w(lnC_{t+1} - lnW_{t+1} + r_{t+1}^w - g_{t+1} - n_{t+1}) + \kappa \tag{1.2}$$

where κ is an unimportant constant in the log-linearized budget constraint. For simplicity, only consider private wealth W_t.

Assume consumption per capita growth (g) and population growth (n) follows a AR(1) process with time-varying volatility $\sigma_{g,t}^2$ and $\sigma_{n,t}^2$. Consumption per capita growth (g) and population growth (n) fluctuate along with first moment shocks (ϵ_g, ϵ_n) and second moment shocks (u_g, u_n), where these shocks exhibit normal distribution with zero mean.

$$g_{t+1} = (1-\rho_g)\mu_g + \rho_g g_t + \epsilon_g, \quad \epsilon_g \sim N(0, (1-\rho_g^2)\sigma_{g,t}^2) \tag{1.3}$$

$$\sigma_{g,t+1}^2 = \alpha_g + \beta_g g_t^2 + \gamma_g \sigma_{g,t}^2 + u_g, \quad u_g \sim N(0, \sigma_{u_g}^2) \tag{1.4}$$

$$n_{t+1} = (1-\rho_n)\mu_n + \rho_n n_t + \epsilon_n, \quad \epsilon_n \sim N(0, (1-\rho_n^2)\sigma_{n,t}^2) \tag{1.5}$$

$$\sigma^2_{n,t+1} = \alpha_n + \beta_n n_t^2 + \gamma_n \sigma^2_{n,t} + u_n, \quad u_n \sim N(0, \sigma^2_{u_n}) \tag{1.6}$$

Deleveraging (ρ) and risk appetite (θ) shock are assumed to follow a AR(1) process with constant variance and innovations to level ϵ_ρ, ϵ_θ.

$$\rho_{t+1} = (1-\rho_\rho)\mu_\rho + \rho_\rho \rho_t + \epsilon_\rho, \quad \epsilon_\rho \sim N(0, \sigma^2_\rho) \tag{1.7}$$

$$\theta_{t+1} = (1-\rho_\theta)\mu_\theta + \rho_\theta \theta_t + \epsilon_\theta, \quad \epsilon_\theta \sim N(0, \sigma^2_\theta) \tag{1.8}$$

Expected risk premium (ERP) is given by

$$ERP_t = E_t r^w_{t+1} - r^f_t = \theta_t \sigma cov_t(r^w_{t+1}, g_{t+1} + n_{t+1}) + (1-\theta_t)\sigma^2_{r,t} \tag{1.9}$$

where $cov_t(r^w_{t+1}, g_{t+1} + n_{t+1})$ is assumed to be small and constant.

Volatility of the return on wealth $\sigma^2_{r,t}$ is assumed to be time-varying as follows:

$$\sigma^2_{r,t+1} = \alpha_r + \beta_r r_t^2 + \gamma_r \sigma^2_{r,t} + u_r, \quad u_r \sim N(0, \sigma^2_{u_r}) \tag{1.10}$$

1.1 Solving the Model

Solve the model using the standard perturbation method, which can be easily implemented by Dynare. Given the initial parameter values, simulate the economy with 1200 periods and drop the first 500 periods to make sure the economy starts from around the steady state value. Repeat the simulation for 100 times. After each simulation, calculate the moments of interest from the model. And then, take average of these moments and match data moments. I describe targeted moments and procedures for the parameterization in the following section.

1.2 Parameterization

First, estimate the GARCH model using real data to obtain information about the parameters for productivity and demographic process, which are mainly determined outside the model.

The estimated coefficients of the GARCH model are plugged into the parameters of equation (1.3) - (1.6). GARCH terms γ_g and γ_n are adjusted to match the observed volatility of consumption per capita growth and population growth, respectively. Next, parameters for deleveraging and risk appetite process, which include persistence (ρ_ρ, ρ_θ) and volatility (σ_ρ, σ_θ), are set to match empirical target moments. A key strategy is to utilize the relations between consumption to wealth (C/W) and risk free rate and risk premium implied by the Euler equation and budget constraint. Deleveraging shock (ϵ_ρ) affects both C/W and risk free rate but does not affect risk premium. Thus, the OLS coefficients β_{rf} and β_{rp} obtained from the regression of risk free rate and risk premium on ln(C/W) contain information in that increasing the persistence or volatility of deleveraging shock increases β_{rf}, while β_{rp} remain almost unchanged. On the other hand, risk appetite shock (ϵ_θ) affects C/W, risk free rate, and risk premium at the same time. Given the deleveraging shock process, persistence or volatility of risk appetite shock is set to match β_{rp}. I also target the volatility of risk free rate and equity premium (σ_{rf} and σ_{rp}) to pin down the volatility of deleveraging shock and risk appetite shock.

Other parameters such as steady state value of risk aversion (γ) are calibrated according to existing literatures. Table 6 compares moments from the model and data.[1] Table 7 summarizes the parameterization of the model.

Table 6: Targeted Moments

symbol	interpretation	model	data (U.S., 1870 - 2015)
σ_{rf}	volatility of risk free rate	4.93	4.93
σ_{rp}	volatility of equity premium	7.57	18.46
σ_g	volatility of consumption per capita growth	3.42	3.42
σ_n	volatility of population growth	0.52	0.52
μ_g	mean of consumption per capita growth	1.7498	1.7498
μ_n	mean of population growth	1.4326	1.4326
β_{rf}	$E_t r^f_{t+1} = \alpha + \beta_{rf} ln(C/W)_t$	0.11	0.13
β_{rp}	$E_t rp_{t+1} = \alpha + \beta_{rp} ln(C/W)_t$	0.04	0.15

Note: Standard deviation and mean are in percentage.

[1] The model fails to match the observed volatility of equity premium and the OLS coefficient β_{rp}. If I increase the volatility of return on wealth shock σ_{u_r}, I can match the data better. σ_{u_r} is assumed to be zero, since it will affect variance decomposition results, and there is no good interpretation for σ_{u_r} in this model.

Table 7: Parameterization

symbol	interpretation	value	target / information
ρ_ρ	persistence of deleveraging shock	0.9	β_{rf}
σ_ρ	volatility of deleveraging shock	0.03	σ_{rf}
ρ_θ	persistence of risk appetite shock	0.5	β_{rp}
σ_θ	volatility of risk appetite shock	2.555	σ_{rp}
μ_g	mean of consumption per capita growth	0.017498	
ρ_g	persistence of productivity shock	0.187	
α_g	constant term of productivity volatility	0.000258	GARCH estimate and σ_g
β_g	ARCH term of productivity volatility	0.259	
γ_g	GARCH term of productivity volatility	0.714	
σ_{u_g}	standard error of productivity volatility shock	0.0344	
μ_n	mean of population growth	0.014326	
ρ_n	persistence of population shock	0.933	
α_n	constant term of population volatility	3.74E-06	GARCH estimate and σ_g
β_n	ARCH term of population volatility	-0.006	
γ_n	GARCH term of population volatility	0.915	
σ_{u_n}	standard error of population volatility shock	0.0019	
α_r	constant term of return on wealth volatility	0.001563	
β_r	ARCH term of return on wealth volatility	0.0091	GARCH estimate
γ_n	GARCH term of return on wealth volatility	0.945	
σ_{u_r}	standard error of return on wealth volatility shock	0	By assumption
κ	constant in the budget constraint	-0.1984	C/W: steady state value = average
ρ_w	constant annual discount rate	1 - 0.0465	ρ_w = 1 - $\exp(ln(C/W)^{ss})$
$1/\sigma$	intertemporal elasticity of substitution	2	
γ	steady state value of risk aversion	2	similar to standard literature
μ_θ	(1-γ)/(1-σ)	-2	
μ_ρ	steady state value of stochastic discount rate	0.03	